Pulsed Laser Cooling for Cavity-Optomechanical Resonators

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A pulsed cooling scheme for optomechanical systems is presented that is able to cool at a much faster rate than possible by conventional methods. The proposed scheme can be implemented for both strongly and weakly coupled optomechanical systems. We discuss analytically its underlying working mechanism, which is based on interferometric control of optomechanical interactions, and we demonstrate its efficiency for pulse sequences that have been obtained with methods from optimal control. The large obtained cooling rates suggest that our scheme provides a significant reduction of current experimental constraints, in particular environment temperature, for optomechanical ground state cooling. Finally, the presented framework can be used to create a rich variety of optomechanical interactions and hence offers a novel, readily available toolbox for fast optomechanical quantum control.

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INTRODUCTION

Micro- and nanomechanical resonators are currently emerging as new quantum systems [1]. Their integrability in a solid state architecture offers attractive opportunities for quantum information objectives such as mechanical quantum registers [2, 3], optomechanical quantum transducers [4] or quantum memories [5]. At the same time, their size and mass promise access to a hitherto untested regime of macroscopic quantum physics [6–10]. A prerequisite to achieve full coherent control over mechanical quantum states is to operate these systems close to their quantum ground state and to achieve coupling rates that exceed all other decoherence rates. The field of cavity quantum optomechanics [11–14] utilizes methods from quantum optics in combination with optomechanical radiation pressure interactions to achieve this and experiments are progressing rapidly. For example, optomechanical cooling close to [15–17, 22] and even well into [18] the quantum ground state of micromechanical devices has been realized. Independently, the strong coupling regime has been demonstrated [19, 20], and optomechanical analogues of electromagnetically induced transparency [21, 22] have shown first steps towards mechanical storage of light.

Notwithstanding these successful developments, the current experimental constraints for full quantum control are quite demanding. The main reason is that most mechanical devices are intrinsically connected to a hot environment through their supports, which results in large heating rates. The most widely used cooling scheme [27, 28] is based on sideband cooling, initially developed for trapped ions [29], whose speed is inherently limited by the trap, i.e. mechanical, frequency ν. In other words, the ultimate cooling rate Γ of such schemes will always be limited through $\Gamma < \nu$. Efficient cooling therefore requires to minimize the thermal coupling, either by operating in a cryogenic cavity [15–18, 23] or by decoupling the mechanical resonator from its environment [24–26]. Alternatively, faster cooling schemes that can beat the mechanical heating rate are required. Recently it was demonstrated in the context of ion trap physics that pulsed schemes can break the speed limit set by the oscillator frequency [30]. The way that these schemes operate is to independently activate either the cooling or the heating process, not by the use of the rotating wave approximation but by interference between different optical pulses incident on the system being cooled. While the validity of the rotating wave approximation poses a natural limit to the strength of interaction, quantum interference does not. This offers a new strategy for fast optomechanical interactions, in particular for optomechanical cooling.

A building block used throughout this work involves the Baker-Campbell-Hausdorff (BCH) formula [31] to analyze a pulse sequence of three segments. In the first segment a pulse $\Omega\hat{o}$ is applied for time t_p, with driving strength Ω and driving term $\hat{o}$. Subsequently, the system is let to evolve freely for a time t_f. Finally, the sequence closes with a counter-pulse $-\Omega\hat{o}$ and duration t_p. We denote this pulse sequence as the BCH transformation $\{-\Omega\hat{o}, 0, \Omega\hat{o}\}$ with a time structure vector (t_p, t_f, t_p). In the analytical derivations made throughout this work the strong pulse limit is assumed, where the pulses are so strong as to allow for the mechanical resonator's free evolution to be neglected. Let us consider the un-pulsed situation described by a Hamiltonian $\hat{h}$. For a commutator $[\hat{o}, \hat{h}]$ commuting with $\hat{o}$, the BCH formula shows that the pulse sequence above is equivalent to transforming the operators of the Hamiltonian $\hat{h}$ to $\hat{h} + \Omega[\hat{o}, \hat{h}]t_p$. A carefully chosen sequence of pulses and counter-pulses can therefore generate, in principle, any linear canonical Hamiltonian and a number of important nonlinear ones [32]. The strength of the desired Hamiltonian is only limited by the strength of the pulses and not by a natural frequency of the system such as the mechanical frequency.

In this letter we demonstrate how the combination of fast pulses with ideas from optimal control allows for efficient and fast cooling of mechanical oscillators. Our scheme utilizes cavity dynamics that proceed faster than the cavity lifetime $1/\kappa$, which is achieved via short pulses and interference between the time-dependent intra- and extra- cavity fields [33]. The pulsed laser sequence creates an approximation to the beam-splitter operator $x_m x_c + p_m p_c \propto ab^\dagger + a^\dagger b$, where a is the annihilation operator of the cavity, b is the annihilation operator of the mechanical oscillator and x, p are the quadrature operators. Use of analytical tools and optimal control, which is a combination of BFGS [34] and simulated annealing [35], are applied to engineer an efficient and fast cooling cycle. The scheme is shown to be applicable in both the good cavity($\frac{\kappa}{\nu} < 1$) and the bad cavity ($\frac{\kappa}{\nu} > 1$) limits. Finally we analyze the efficiency of the scheme and discuss its experimental feasibility.

PHYSICAL SETTING

Cavity optomechanical systems fundamentally involve an optical cavity field which couples to a mechanical resonator due to radiation pressure. Here the optical (mechanical) mode, oscillating at a frequency ω (ν) is characterized by a relaxation timescale κ (γ_m). The optical mode is driven by a detuned laser field of frequency ω_l with a strength Ω. The Hamiltonian corresponding to this system reads:

$$H = \Delta a^\dagger a + \nu b^\dagger b + \frac{g_0}{\sqrt{2}} a^\dagger a(b^\dagger + b) + \Omega(a^\dagger e^{-i\phi} + a e^{i\phi}) \quad (1)$$

where ϕ is the initial phase of the driving, $\Delta = \omega_l - \omega$ and g_0 is the optomechanical coupling rate between oscillator and cavity modes.

LINEAR APPROACH

The complete optimization in the non-linear regime is challenging. Hence we are now breaking the problem down into more readily treatable units. First, in the limit where $g_0 \ll \nu$, we consider a linearized problem for which optimization is straightforward. A pulse sequence is proposed in this section that efficiently adds an anti-Stokes operator to the Hamiltonian. The so determined coupling rates will determine the required cavity photon number and will thus constitute a first approximation to the required laser pulse sequence. This sequence can then serve as a starting point for the final optimization in the fully non-linear setting.

Since this scheme makes use of rapidly changing driving, the usual procedure [19, 27] to linearize the Hamiltonian in eq. (1) cannot be easily applied. It is necessary to move to a frame co-moving with the state of the cavity. This will correspond to an interaction picture with respect to the non coupled part of the cavity Hamiltonian. In this frame the operator a is by definition a small perturbation generated by the weak coupling parameter g_0, and this allows to drop the quadratic term of the coupling. The definition $x_j = \frac{1}{\sqrt{2}}(a_j + a_j^\dagger)$ and $p_j = \frac{i}{\sqrt{2}}(a_j^\dagger - a_j)$ for any mode j allows for a compact expression of the Hamiltonian:

$$H = \Delta a^\dagger a + \nu b^\dagger b + (G(t)a + G^*(t)a^\dagger)x_m + |G(t)|^2 x_m \quad (2)$$

Where $G(t) = ig_0 e^{-i(\Delta - i\kappa)t} \int_0^t \Omega(t') e^{i(\Delta - i\kappa)t'} dt'$ is the time integral of the driving power, the subscript m stands for the mechanical mode and the subscript c for the cavity mode is used below.

The linear behavior of the dynamics admits the use of the covariance matrix approach [36]. Decoherence effects from the coupling to the environment can easily be monitored. We define the vector $\mathbf{R} \equiv (x_c, p_c, x_m, p_m)^t$ so that the matrix of covariances reads $\gamma_{i,j} \equiv 2Re\{\langle R_i R_j \rangle - \langle R_i \rangle \langle R_j \rangle\}$. Its equation of motion and the matrices defining it are:

$$\frac{d}{dt}\gamma = M\gamma + \gamma (M)^T + \frac{\kappa}{2} P \quad (3)$$

$$M = SV - \frac{\kappa}{2} P \quad (4)$$

$$P = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

where V is the potential matrix and S is the symplectic matrix.

Let's now consider the BCH transformation $\{-Gp_c x_m, 0, Gp_c x_m\}$ of time vector (t_1, t_f, t_1), where G is the absolute value of the function $G(t)$. Only the operators x_c and p_m will be affected in the Hamiltonian, since they are the only ones for which the commutator with the pulse operator is not zero. Since $[x, p] = i$, this transformation is equivalent to the substitutions $p_m \to p_m + Gt_1 p_c$ and $x_c \to x_c - Gt_1 x_m$, thus obtaining the following Hamiltonian:

$$H = H_0 + 2Gt_1 \nu p_c p_m - 2Gt_1 \Delta x_c x_m. \quad (6)$$

where H_0 is the Hamiltonian in eq.(2) for $G = 0$. A further $4Gt_1 \Delta x_c x_m$ pulse can compensate for the sign difference of the coefficients of both terms. This way, under the resonance condition $\Delta = \nu$, a beam-splitter (red sideband) operator has been generated [37]. In a similar way, without a compensating pulse, a blue-sideband operator can be generated ($x_c x_m - p_c p_m$). The requirements on the laser power for this sequence to work can be estimated to $\Omega \gg 10\nu^2/g_0$, eq. (6), assuming one tenth of the pulse time is allowed for the control variable to adopt a new value. This requirement can be considerably relaxed by the use of optimal control methods which also yields higher cooling rates.

Optimizations Using Optimal Control

Optimizations have been performed using QLib's [38] optimization infrastructure, which interchanges hill-climbing with simulated annealing stages, and run on the University of Ulm CUSS cluster. We have taken multiple approaches to the optimizations presented here: (a) initially optimizing on the pulse amplitude only and subsequently optimizing both amplitude and phase; (b) using the analytically derived sequence as an initial point of the optimization (with parametrization possibly extended to multiple applications of the theoretical sequence, all of which is then Trotterized); (c) "pushing" sequences to shorted times and / or higher dissipating cavities by a series of optimizations with increasingly constraining parameters, where the result of optimization n serves as the initial conditions for optimization $n+1$ and, of trivially (d) random starting conditions and simultaneous optimization of all control parameters.

An example of such an optimal sequence has been obtained using the full range of interactions, $(Re[G]x_c + Im[G]p_c)\,x_m$, and is presented in fig. (1). Starting with an initial phonon occupation of 100, and for $\kappa = 0$, we are able to achieve a final occupation below 2×10^{-7} in 0.57 times the period of the mechanical oscillator.

The cooling performance for a wide set of cooling sequences is presented in fig. (2) - results of several covariance matrix optimizations for varying values of the cavity decay rate κ, and compared to the results of the analytical sequence without numerical optimization. All sequences take at most $0.8 \times 2\pi/\nu$ to reach their lowest phonon number. All of the sets are initialized with 10 phonons and allow for a maximal increase in the coupling control of 10ν. In all cases the optimal control method obtains better results than the initial guess. Moreover it can be seen that when the analytical pulse (eq. 6) was given as an initial guess the final optimization results were significantly better.

NON-LINEAR CASE

For systems with a strong coupling parameter g_0 [39, 40], it is necessary to treat the full nonlinear description of the coupling presented in eq.(1). Without further modifications, this system provides a picture where the BCH approach used above generates undesired terms that cannot be neglected. The fact that the cavity cou-

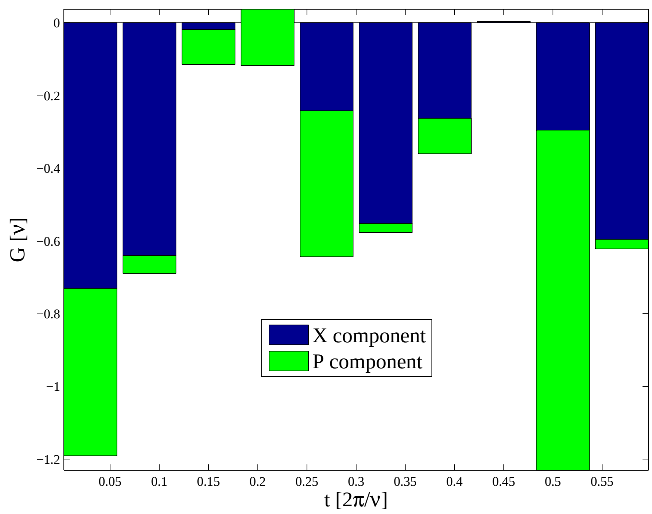

FIG. 1. Sample pulse sequence optimized for the full linear interaction, with initial phonon occupation of 100, $G_{max} = \nu$, achieving a final occupation below 2×10^{-7}.

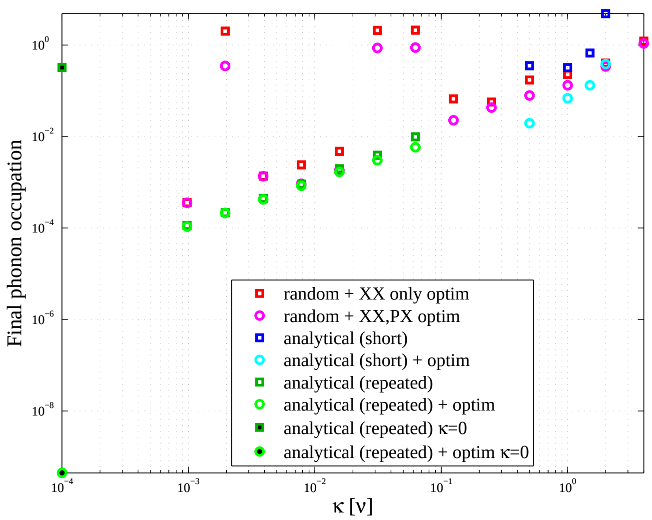

FIG. 2. Results of several covariance matrix optimization procedures. All of them have an initial phonon occupation of 10, $G_{max} = 10\nu$. The red and magenta data sets involve optimizations with random initial pulse-sequences, optimized with partial ($x_c x_m$-only) and full coupling respectively. Final temperatures achieved by sequences of 30 pulses (based on the four pulse analytical formulations, repeated 7.5 times) are shown before and after optimization (blue and cyan). Long cyclic sequences (300 pulses, 75 times the 4 pulse analytical sequence) are shown before and after optimization and are represented by the dark and light green sets. A repetition of the last set before optimization for $\kappa = 0$ appears at the left axis of the plot.

ples quadratically to the mechanical oscillator makes the perturbations insensitive to sign changes, so that the previously given pulse/counter-pulse picture does not help to overcome the undesired terms.

First let us analyze the effect that an analytical sequence equivalent to that applied in the linear case would have in the non-linear approach. In this case a $p_c x_m$ pulse operator cannot be produced straight away, though, but only after a BCH subsequence $\{-\Omega x_c, 0, \Omega x_c\}$. Already this introduces a mechanical driving of the form $g_0(\Omega t_p)^2 x_m$ that cannot be countered by any sort of cavity driving. The mechanical operators need then to be displaced, otherwise higher order terms would be introduced.

A more delicate issue is the one associated to pulses of pairs of operators involving both the cavity and mechanical oscillator. This generates cubic terms in the position quadrature of the oscillator that renders it unstable. This is hence the case when applying for a time t_1 a nested BCH transformation for the pulse $2g_0\Omega t_p p_c x_m$ obtained during the subsequence. As shown for the linear case, this is necessary in order to obtain the beam-splitter operator, and will imply the two term substitutions $x_c \to x_c - \Omega x_m$ and $p_m \to p_m + \Omega p_c$. Defining $\Omega' \equiv 2g_0\Omega t_p$ the effective Hamiltonian then reads:

$$\begin{aligned} H = H_0 &+ 2\nu\Omega' t_1 p_c p_m - 2\Delta\Omega' t_1 x_c x_m \\ &+\nu(\Omega' t_1)^2 p_c^2 + \Delta(\Omega' t_1)^2 x_m^2 \\ &-2g_0\Omega' t_1 x_c x_m^2 + g_0(\Omega' t_1)^2 x_m^3. \end{aligned} \tag{7}$$

The terms in the first line correspond to the beam-splitter operator when $\Delta = \nu$. The second line presents quadratic terms, for which a Bogoliubov transformation would be in order to absorb them to the frequency terms of the original Hamiltonian (see appendix A). The cubic terms in the last line are proportional to g_0. In the case of systems with $g_0 \ll \nu$, these terms can be neglected and hence the linear approximation survives. In the regime treated in this section, however, g_0 is not a small parameter and the cubic terms cannot be neglected.

Further modifications to the system are hence required in order to tackle this situation. A double cavity approach is proposed [41–43] where the coupling to the oscillator is of opposite sign for each mode. In Fabry-Perot cavity settings, this corresponds to using the back of the oscillator as a mirror for a second cavity, the layout is presented in fig. (3). On the one hand, this cancels the mechanical oscillator driving by the opposite effect of the radiation pressures of each cavity. On the other hand, it allows to distribute the coupling to the oscillator between the two modes so that none of the two cavity modes is coupled quadratically to the mechanical mode. The following is the Hamiltonian of the system without driving:

$$H_0 = \Delta(a_1^\dagger a_1 + a_2^\dagger a_2) + \nu b^\dagger b + \frac{g_0}{\sqrt{2}}(a_1^\dagger a_1 - a_2^\dagger a_2)(b^\dagger + b), \tag{8}$$

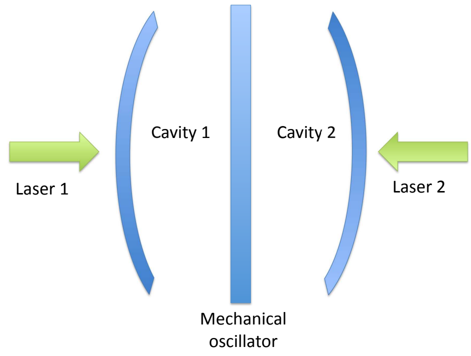

FIG. 3. Layout of the double cavity plus resonator system. The fact that the cavity is accessed from opposite directions makes the coupling to the oscillator be of opposite sign.

where the subscript distinguishes the two cavity modes and Δ is the detuning for both sides of the double cavity. As discussed above, the pulsing rate is assumed to be larger than the decay rate of the cavity modes and the objective is to beat the heating rate of the resonator. Therefore, the incoherent dynamics will be neglected in this section. The definition of symmetric and antisymmetric modes simplifies the analytical procedures:

$$\begin{aligned} a_a &= \frac{1}{\sqrt{2}}(a_1 + a_2) \\ a_s &= \frac{1}{\sqrt{2}}(a_1 - a_2) \end{aligned} \tag{9}$$

so that the Hamiltonian becomes:

$$H_0 = \Delta(a_a^\dagger a_a + a_s^\dagger a_s) + \nu b^\dagger b + g_0(x_a x_s + p_a p_s)x_m. \tag{10}$$

It is possible to drive the symmetric or the antisymmetric mode and then effectively engineer the linear transformation between the mechanical oscillator and the other mode. After the full sequence is implemented, a Trotter decomposition is useful to counter undesired driving terms of the cavity.

The BCH transformation $\{-\Omega x_a, 0, \Omega x_a\}$ would generate the following Hamiltonian:

$$H = H_0 + g_0\Omega t_1 p_s x_m + 2\Delta\Omega t_1 p_a. \tag{11}$$

Instead of restricting to free evolution between the two pulses, it is useful to also drive the cavity during this time segment with a p_a pulse so that the coefficient of the last term can be controlled at will. By defining this pulse as $-(\alpha + 2)\Delta\Omega t_1 p_a$ the last driving ends up becoming:

$$H = H_0 + g_0\Omega t_1 p_s x_m - \alpha\Delta\Omega t_1 p_a. \tag{12}$$

The following list denotes the proposed driving sequence for the cavity, where the notation indicates a nested BCH transformation:

$$\{\{-\Omega x_a, \beta p_a, \Omega x_a\}, 0, \{\Omega x_a, \beta p_a, -\Omega x_a\}\}, \tag{13}$$

where $\beta \equiv -(\alpha+2)\Delta\Omega t_1$ and the time vector is:

$$((t_1, t_f, t_1), t'_f, (t_1, t_f, t_1)) \tag{14}$$

with $t_2 \equiv t_1 + t_f + t_1$ and $t_3 \equiv 2t_2 + t'_f$.

Taking $\Omega t_1 \gg 1$, H_0 in eq.(12) can be neglected. The inverted transformation $\{\Omega x_a, 0, -\Omega x_a\}$ together with the inverted cavity driving during the free evolution will generate the same expression with opposite sign, so that these highest order terms can be interpreted as a pulse in a further BCH transformation. This nested transformation of pulse duration t_2 results in the substitutions $x_a \to x_a + \alpha\Delta\Omega t_1 t_2$, $x_s \to x_s - g_0\Omega t_1 t_2 x_m$ and $p_m \to p_m + g_0\Omega t_1 t_2 p_s$. For simplicity the definition $\tau_1 \equiv \Omega t_1 t_2$ is used:

$$\begin{aligned} H = H_0 &+ (\alpha-2)g_0\Delta\tau_1 x_s x_m + 2g_0\nu\tau_1 p_m p_s \\ &+\nu g_0^2\tau_1^2 p_s^2 - 3\Delta g_0^2\tau_1^2 x_m^2 - \tau_1 g_0^2 x_a x_m^2 \\ &+2\alpha\tau_1\Delta^2 x_a. \end{aligned} \tag{15}$$

This Hamiltonian presents an $x_s x_m$ and a $p_s p_m$ term with the same coefficient if the resonance condition $\Delta = 2\nu/(\alpha-2)$ is met. For a driving where $\alpha = 4$ the resonance condition is $\Delta = \nu$, which is important in experimental layouts where the strength of the beam-splitter operator would be limited to a value of order ν in order to facilitate full energy transfer. In the end of the sequence a x_a pulse of area $-2\alpha\tau_1 t_3\Delta^2$ can counter the last term in eq.(15). In order for this to be as effective as possible it is advisable to perform a Trotter decomposition between the full sequence and the driving.

The rest of the terms are negligible: the one proportional to x_a is kept to a minimum thanks to the counter-driving of x_a (as mentioned at the end of the last paragraph), and the two quadratic terms can be absorbed into the corresponding frequency terms by means of a Bogoliubov transformation (see appendix A). This transformation leaves the $x_s x_m + p_s p_m$ operator invariant. Nevertheless it is worth pointing out that quadratic terms are proportional to $g_0^2\tau_1^2$, i.e., they are only going to be relevant for high laser powers. In order to cool faster than the trap frequency the following inequality has to be satisfied: $\Omega g_0 > 10^2\nu^2$ (when all the t_i's are of the same order of magnitude and are taken to be one order of magnitude smaller than the mechanical oscillation period). This requirement results from eq. 15 by demanding that the pre-factor of the beam-splitter operator ($x_s x_m + p_s p_m$ term) equals ν. As compared to the values obtained for the linear approach, its requirements are less challenging for the reason that the control variable is here a direct physical parameter and not a time integral of it, so sudden changes in the control variable translate to very steep increases in the physical control. Finally, note that in the non-impulsive limit (i.e. when $\Omega x \gg H_0$ is false), as the free Hamiltonian is present while pulsing, one may do without the periods of free evolution, as all terms required for the commutation relations to generate the cooling operator are present; pre-factors in this case will be obviously different.

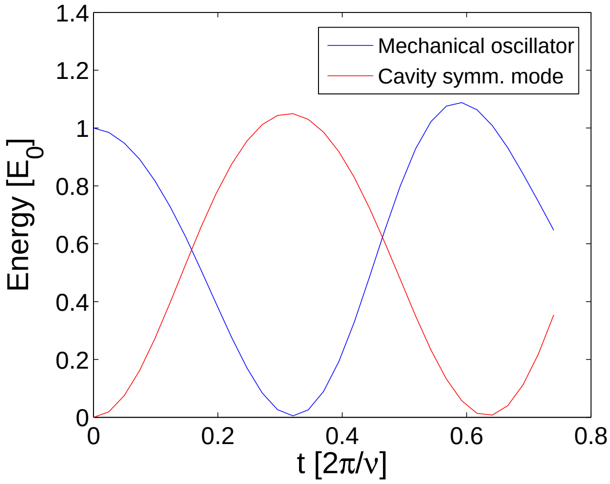

FIG. 4. Energy of the mechanical oscillator and the symmetric cavity mode as a function of time.

We have analyzed the effects of the nonlinear terms via numerical simulation of the presented pulse sequence. Figure (4) shows the resulting coherent state swapping between the mechanical oscillator and the symmetric cavity mode - a complete anti-Stokes process.

EXPERIMENTAL FEASIBILITY AND VERIFICATION

We will finally discuss the feasibility of our pulsed scheme for current optomechanical systems. To obtain a laser cooling rate Γ beyond the limitation of continuous sideband-cooling, i.e. $\Gamma > \nu$, readily available experiments are sufficient: for an optomechanical Fabry-Perot cavity one can easily obtain $\nu = 2\pi 10^6$ Hz, $m_{eff} = 5\times10^{-11}$ kg and $\kappa = 0.75\nu$ [16, 19], which yields $g_0 = 75$ Hz $\ll \nu$ and hence satisfies the linear regime of pulsed laser cooling (we have assumed a cavity length $l = 10^{-2}$ m and an optical pump wavelength $\lambda = 1064$ nm). Typical cooling rates that have been achieved thus far in this regime are on the order of Γ $10^{-1}\times\nu$ [16]. In contrast, by using optimal control we can design a 10-pulse cooling sequence of total duration $0.75\,\frac{2\pi}{\nu}$ and pulse energies of ≈ 40 nJ per pulse to obtain a cooling rate $\Gamma = 1.3\,\nu$. Since here each pulse lasts $0.075\mu s$, such pulse energies can by created directly from amplitude modulating an $0.5W$ continuous laser beam. For an initial temperature of $T = 1K$ and $\gamma_m \approx 300$Hz ($Q = 2 \times 10^4$) this sequence will already reach mechanical thermal occupancies on the order of $n_f \approx 0.1$.

As a second example we consider an optomechanical Fabry-Perot double microcavity with $\nu = 2\pi 10^4$ Hz, $m_{eff} = 10^{-10}$ kg, $\kappa = 2 \times 10^5 \nu$ and individual cavity lengths $L_1 = L_2 = 4\lambda$, as has been suggested in [44]. The resulting $g_0 = 10^6$ Hz $> \nu$ satisfies the nonlinear regime of pulsed laser cooling. Using the analytically derived BCH sequence, our method requires a set of laser pulses of length $\ll 50$ ps with a maximal peak power of 1 kW, which is a readily available technology for example in form of Q-switched lasers. This will result in a net cooling rate $\Gamma = 10^4 \nu$. For $\gamma_m \approx 1$, i.e. a Q-factor of 6×10^4, this would already allow cooling to the quantum ground state starting from room temperature.

CONCLUSIONS

We have introduced a novel pulsed cooling method for mechanical oscillators, which surpasses the intrinsic limit of conventional continuously pumped cooling. Our scheme is based on generating a specific optomechanical interaction, here the beam-splitter (cooling) interaction, by quantum interference of successive pulses. While already a simple analytical approach provides otherwise unachievable cooling rates $\Gamma > \nu$, the use of optimal control methods can significantly enhance these rates even further. We have also shown that already current optomechanical configurations could achieve dramatic improvements in their experimental performance. In a similar way as presented in this work it is possible to generate a rich class of optomechanical interactions, for example the downconversion (entangling) interaction $x_c x_m - p_c p_m$ or various non-linear terms. This establishes a new and complete tool kit for fast preparation and manipulation of optomechanical quantum states and may very well provide a route towards room temperature quantum optomechanics.

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While finishing this paper we learnt of reference [45], which also treats pulsed cooling schemes for optomechanical systems.

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Appendix A

In this section the values of a general Bogoliubov transformation are derived so that any quadratic term $\alpha x^2 + \beta p^2$ is transformed into a frequency term $\delta(x'^2 + p'^2)$. If the transformation between modes a and b is defined as

$$\begin{aligned} b &= ua + va^\dagger \\ b^\dagger &= u^* a^\dagger + v^* a \\ |u|^2 - |v|^2 &= 1 \\ a &= u^* b - v b^\dagger \\ a^\dagger &= u b^\dagger - v^* b \end{aligned} \tag{16}$$

the effect on a general quadratic term will be

$$\begin{aligned} \Delta(a^\dagger a + \frac{1}{2}) + \Omega p^2 = \Delta(x^2+p^2) + \Omega p^2 = \alpha x^2 + \beta p^2 \\ = \left(\sqrt{\alpha}x - i\sqrt{\beta}p\right)\left(\sqrt{\alpha}x + i\sqrt{\beta}p\right) + \sqrt{\alpha\beta} \\ = 2\sqrt{\alpha\beta}\left[\frac{1}{2}\left(\sqrt[4]{\frac{\alpha}{\beta}}x - i\sqrt[4]{\frac{\beta}{\alpha}}p\right)\left(\sqrt[4]{\frac{\alpha}{\beta}}x + i\sqrt[4]{\frac{\beta}{\alpha}}p\right) + \frac{1}{2}\right] \\ = \Delta'\left(b^\dagger b + \frac{1}{2}\right) \end{aligned} \tag{17}$$

where $[x,p] = i$. Now the parameters of the Bogoliubov transformation can be found

$$\begin{aligned} b = \frac{1}{\sqrt{2}}\left(\sqrt[4]{\frac{\alpha}{\beta}}x + i\sqrt[4]{\frac{\beta}{\alpha}}p\right) \\ = \frac{1}{2}\left(\sqrt[4]{\frac{\alpha}{\beta}} + \sqrt[4]{\frac{\beta}{\alpha}}\right)a + \frac{1}{2}\left(\sqrt[4]{\frac{\alpha}{\beta}} - \sqrt[4]{\frac{\beta}{\alpha}}\right)a^\dagger \\ = \frac{1}{2}\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt[4]{\alpha\beta}}a + \frac{1}{2}\frac{\sqrt{\alpha}-\sqrt{\beta}}{\sqrt[4]{\alpha\beta}}a^\dagger \end{aligned} \tag{18}$$

so that:

$$\begin{aligned} u &= \frac{1}{2}\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt[4]{\alpha\beta}}, \\ v &= \frac{1}{2}\frac{\sqrt{\alpha}-\sqrt{\beta}}{\sqrt[4]{\alpha\beta}}. \end{aligned} \tag{19}$$

The effect of this transformation on linear terms is

$$\begin{aligned} x = a + a^\dagger = u^* b - v b^\dagger + u b^\dagger - v^* b \\ = (u^* - v^*)b + (u - v)b^\dagger = Re(z_-)x' + Im(z_-)p' \\ p = i(a^\dagger - a) = i(u b^\dagger - v^* b - u^* b + v b^\dagger) \\ = i((u+v)b^\dagger - (u^* + v^*)b) = Re(z_+)p' - Im(z_+)x' \end{aligned} \tag{20}$$

where:

$$\begin{aligned} z_- &= \sqrt[4]{\frac{\beta}{\alpha}}, \\ z_+ &= \sqrt[4]{\frac{\alpha}{\beta}}. \end{aligned} \tag{21}$$

Appendix B

A differential equation for the amplitude of the field in the cavity $E_{in}(t)$ as a function of a variable external field $E_{out}(t)$ of frequency ω is derived here. First of all, a recursion formula can be found by considering the state of the field inside the cavity after a round trip time $\tau \equiv \frac{2L}{c}$, with L the length of the cavity. It can be considered to be the sum of (a) the previous field after bouncing off the mirrors of each end of the cavity (of reflectivity coefficient $\sqrt{R}$) and having picked up the phase $e^{i\omega\tau}$ and (b) the field that has entered the cavity during this time through the front quarter-wavelength mirror of transmissivity $\sqrt{T}$. This can be expressed as:

$$E_{in}(t+\tau) = Re^{i\omega\tau}E_{in}(t) + i\sqrt{T}E_{out}(t+\tau). \quad (22)$$

The detuning $\Delta = \omega - \omega_c$, with $\omega_c = \frac{\pi}{L}c$, provides us with the only relevant phase difference:

$$E_{in}(t+\tau) = Re^{i\Delta\tau}E_{in}(t) + i\sqrt{T}E_{out}(t+\tau). \quad (23)$$

Assuming that the driving laser is almost in resonance with the cavity eigenfrequency($\Delta\tau \gg \frac{2\pi}{\omega}$) the phase term can be Taylor expanded to first order:

$$E_{in}(t+\tau) = R(1+i\Delta\tau)E_{in}(t) + i\sqrt{T}E_{out}(t+\tau). \quad (24)$$

For lossless cavities $R = 1 - T \approx 1$, and therefore $T\Delta\tau$ terms can be neglected:

$$\begin{aligned} E_{in}(t+\tau) = {} & E_{in}(t) - TE_{in}(t) \\ & + i\Delta\tau E_{in}(t) + i\sqrt{T}E_{out}(t+\tau). \end{aligned} \quad (25)$$

In the limit where the dynamics in the cavity are negligible during the short τ timescale, a differential equation can be derived:

$$\begin{aligned} \frac{E_{in}(t+\tau) - E_{in}(t)}{\tau} = {} & -\frac{T}{\tau}E_{in}(t) \\ & + i\Delta E_{in}(t) + i\frac{\sqrt{T}}{\tau}E_{out}(t+\tau). \end{aligned} \quad (26)$$

Since $\kappa = \frac{T}{\tau}$:

$$\frac{dE_{in}}{dt} = -\kappa E_{in}(t) + i\Delta E_{in}(t) + i\sqrt{\kappa}\frac{1}{\sqrt{\tau}}E_{out}. \quad (27)$$

It is possible to relate this equation to the regular definition for the driving:

$$\Omega = \sqrt{\frac{\kappa P}{\hbar\omega}}. \quad (28)$$

The power P of an electromagnetic wave can be further expressed in terms of the field amplitude E as $P = \frac{Energy}{time} = \frac{\epsilon_0 E^2 Acdt}{dt} = \epsilon_0 E^2 Ac$, where A is the area of the cross section of the laser and c the speed of light. Thus:

$$\Omega = \sqrt{\frac{\kappa P}{\hbar\omega}} = \sqrt{\frac{\kappa\epsilon_0 E_{out}^2 A2L}{\hbar\omega\tau}} = \sqrt{\frac{\epsilon_0 V}{\hbar\omega}}\sqrt{\frac{\kappa}{\tau}}E_{out}. \quad (29)$$

The pre-factor $\sqrt{\frac{\epsilon_0 V}{\hbar\omega}}$ can be expressed as $\sqrt{\frac{\epsilon_0 E^2 V}{\hbar\omega}}\frac{1}{E} = \frac{\sqrt{n}}{E}$, so that it can be interpreted as the inverse of the electric field associated with one photon. The eq.(27) can be rewritten as:

$$\begin{aligned} \sqrt{\frac{\hbar\omega}{\epsilon_0 V}}\frac{dE_{in}}{dt} = {} & -\kappa\sqrt{\frac{\hbar\omega}{\epsilon_0 V}}E_{in}(t) \\ & + i\Delta\sqrt{\frac{\hbar\omega}{\epsilon_0 V}}E_{in}(t) + i\Omega \end{aligned} \quad (30)$$

which is by definition:

$$\frac{da}{dt} = -\kappa a + i\Delta a + i\Omega \quad (31)$$

with a the field amplitude inside the cavity.

An alternative route to obtain the same result is to transform the square of the last term in eq.(27) into:

$$\frac{E_{out}^2}{\tau} = \frac{n(t)}{\tau}\frac{E_{out}^2}{n(t)} = \frac{U}{\tau}\frac{1}{\hbar\omega}\frac{E_{out}^2}{n(t)} = \frac{P}{\hbar\omega}\frac{E_{out}^2}{n(t)}. \quad (32)$$

So the equation has to be rewritten as:

$$\frac{dE_{in}}{dt} = -\kappa E_{in} + i\Delta E_{in} + i\sqrt{\frac{\kappa P}{\hbar\omega}}\frac{E_{out}}{\sqrt{n}}. \quad (33)$$

In order to get an equation for the field amplitudes, one has to divide by the field amplitude carried by one phonon ($\frac{E}{\sqrt{n}}$):

$$\frac{da}{dt} = -\kappa a + i\Delta a + i\sqrt{\frac{\kappa P}{\hbar\omega}}. \quad (34)$$